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TITLE DIMENSION DENSITY - AN INTENSIVE MEASURE OF CHAOS IN  
SPATIALLY EXTENDED TURBULENT SYSTEMS

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**Title: Dimension Density - An Intensive Measure Of Chaos  
In Spatially Extended Turbulent Systems**

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# Dimension density - an intensive measure of chaos in spatially extended turbulent systems

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The determination of correlation dimensions by the Grassberger-Procaccia algorithm from an experimental time series has become a standard tool in the analysis of low dimensional chaotic systems [1]. Here we want to carry over this method to spatially extended systems which have a decaying spatial correlation. In these cases the total number of degrees of freedom or overall "dimension" grows with the size of the system. Then in a finite size system the dimension of the overall dynamics can be recovered already from a single point measurement, if the resolution is greater than some size dependent threshold. Therefore we expect that the measured dimension values will increase when smaller and smaller spatial structures are resolved. This feature is also observed in turbulence experiments [2]. Thus the objective is to get an intensive (i.e. size independent) measure which locally characterizes turbulent systems.

Following an idea of Y. Pomeau [3], we study the *dimension density* or density of degrees of freedom with the help of one dimensional coupled map lattices described by a quantity  $u(x_n, t_n)$  (see [4] for a review). When we consider a fixed resolution  $\epsilon_0$  then we expect that the dynamics  $u(x_0, t)$  measured at one point  $x_0$  will be influenced by the dynamics of points in a neighborhood  $U(x_0, \epsilon_0)$ . The dimension  $D_2(x_0)$  measured for  $u(x_0, t)$  should then be determined by the dynamics of  $U(x_0, \epsilon_0)$ .

The interdependence between two signals at  $x_1$  and  $x_2$  can be measured via the *two-point dimension*  $D_2^{(2)}(x_1, x_2)$ . It shall be defined as the correlation dimension of a combined time series with contributions from the two signals  $u(x_1, t)$  and  $u(x_2, t)$  (e.g. by interleaving the series or by adding them). If the dynamics at the two points is fully independent, we get  $D_2^{(2)}(x_1, x_2) = D_2(x_1) + D_2(x_2)$  as one can easily see. On the other hand, we have  $D_2^{(2)}(x_1, x_1) = D_2(x_1)$ . Furthermore we expect  $D_2^{(2)}(x_1, x_1 + \Delta)$  to be a continuous, monotonously increasing function of the separation  $\Delta$ . The dimension density  $\rho(x_0)$  is then defined as the *rate of change* of the two-point dimension at  $x_0$ :

$$\rho(x_0) = \lim_{\Delta \rightarrow 0} \frac{D_2^{(2)}(x_0, x_0 + \Delta) - D_2(x_0)}{\Delta} \quad (1)$$

The dynamics at each lattice point is generated by a "tent" map  $h(u) = 1 - 2|u - 0.5|$ . The coupling is diffusive with nearest-neighbor interaction:

$$u_{n+1}^{(i)} = h(u_n^{(i)} + \kappa(u_n^{(i-1)} - 2u_n^{(i)} + u_n^{(i+1)}) + \eta(i, n)) \quad (2)$$

where  $u_n^{(i)}$  denotes the function value at the  $i$ -th lattice site at time step  $n$ ,  $\kappa$  the coupling parameter and  $\eta$  an additional noise term. In our numerical computations we use a lattice of 100 maps with

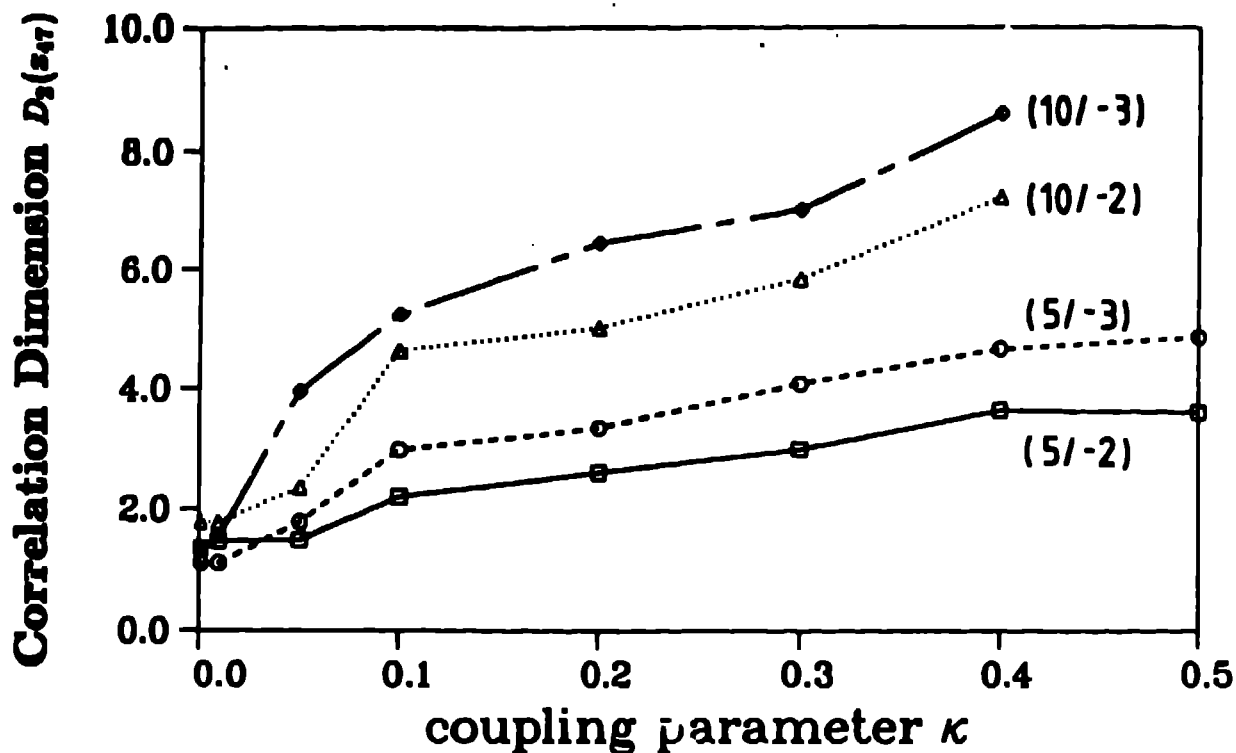


Fig. 1 Correlation dimension measured from single time series at reference point  $x_{47}$  at noise level  $\eta = 10^{-6}$ . The embedding dimension  $D$  and resolution of measurement  $\log_2(\epsilon_0)$  are given in parentheses.

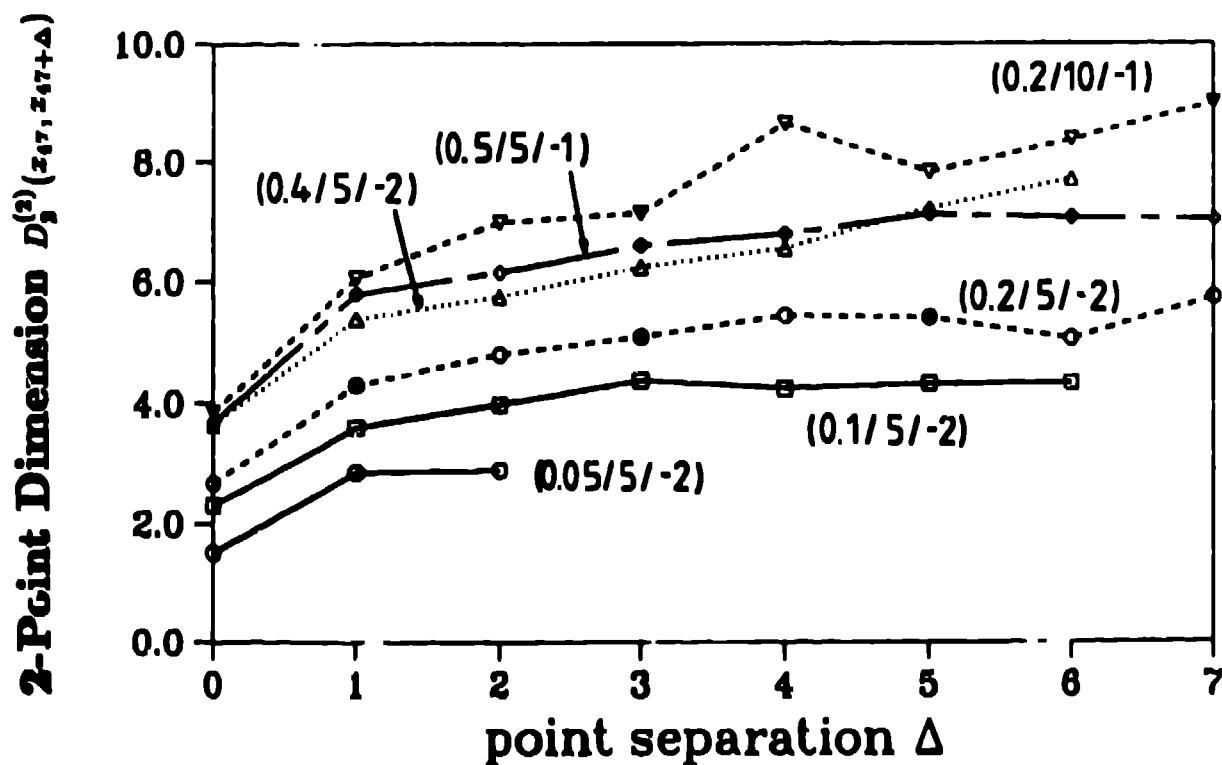


Fig. 2 Two-point dimension of points  $x_{47}$  and  $x_{47+\Delta}$  as a function of separation  $\Delta$ . The corresponding values of coupling strength  $\kappa$ , embedding dimension  $D$  and resolution  $\log_2(\epsilon_0)$  are given in parentheses.

the coupling parameter  $\kappa$  vary between  $10^{-4}$  and 0.5, beyond which the lattice gets unstable. While a single tent map causes numerical problems by hitting the origin after a few iterations, this has not been observed here. We compute  $D_2^{(2)}$  of  $\{u_n^{(0)}\}$  and  $\{u_n^{(i+\Delta)}\}$  by analyzing the series of vectors  $\begin{pmatrix} u_n^{(0)} \\ u_{n+\Delta}^{(i)} \end{pmatrix}$  and embedding them in the corresponding  $2D$  dimensional reconstructed phase space (typically we use about  $n = 10000$  points and  $D \leq 20$ ).

The numerical analysis of (2) shows that the standard methods have to be applied with great care. The correlation graphs have in general - as had to be expected - a slope which becomes steeper at small distances. Furthermore they do not converge with increasing embedding dimension for small  $\epsilon_0$ . Thus the correlation dimension in a strict sense can not be proven to be finite. Therefore we take the local slope of the correlation graphs for fixed embedding dimension and fixed  $\epsilon_0$  as a measure of correlation. It is well known that the method of Grassberger and Procaccia yields large errors for high correlation dimensions [1]. This imposes a limit on the accuracy of our results, especially if only small amounts of data are available. Since our model is spatially discrete, we replace the limes in definition (1) by a finite difference. To compute  $\rho$  at a reference point  $x_0$  we determine the two quantities  $D_2(x_i)$  and  $D_2^{(2)}(x_i, x_i + \Delta)$ ,  $\Delta = 0, 1, 2, \dots$ . Fig. 1 shows the slope of the correlation graphs for measurements of  $D_2(x_{47})$  for various values of  $\kappa$ , embedding dimension  $D$  and resolution  $\epsilon_0$  (at a fixed noise level of  $10^{-6}$ ). It can be clearly seen, that  $D_2$  rises with increasing coupling strength, embedding dimension and resolution (i.e. smaller  $\epsilon_0$ ). The given values for small  $\epsilon_0$  and high  $D$  are rather unreliable, however. In Fig. 2 we have plotted the two-point dimension versus point separation for different values of  $\kappa$ ,  $D$  and  $\epsilon_0$  and the same  $\eta$ .  $D_2^{(2)}$  shows up as a generally increasing function of  $\Delta$  which saturates for large distances at approximately  $2D_2(x_{47})$ . The fluctuations in the displayed curves can probably be explained with the large statistical errors of our measurements.

This result is in full agreement with the expectation that the two-point dimension should be the sum of the single-point dimensions if the two points are independent. Fig. 2 also shows that the two-point dimension increases more rapidly for small  $\Delta$  than for larger ones. This makes a reliable estimation of the dimension density for high  $\kappa$  very difficult.

A more detailed discussion, which also includes the mutual information content between two points, separated in space and time will be presented elsewhere [5].

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